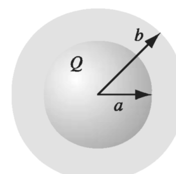
# Electricity and Magnetism, Exam 3, 05/04/2019

5 questions, 75 points total

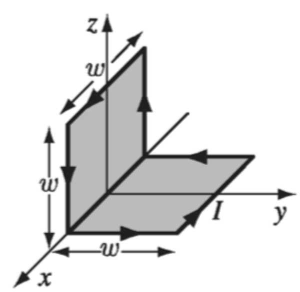
Write your name and student number on the answer sheet. Use of a calculator is allowed. You may make use of the provided formula sheet. The same notation is used as in the book, i.e. a bold-face $\mathbf{A}$ is a vector, $\hat{\mathbf{x}}$ is the unit vector in the x-direction, and $T$ is a scalar.

1. A metal sphere of radius $a$ carries a charge $Q$ (see figure on the right). It is surrounded, out to a radius $b$, by linear dielectric material of permittivity $\epsilon$.



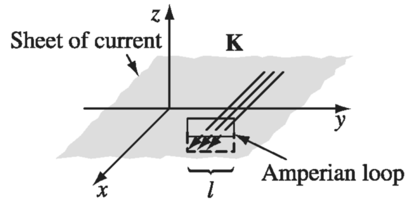

   (a) (5 points) Calculate the displacement $\mathbf{D}$ for all points $r > a$.
   (b) (5 points) Calculate the electric field $\mathbf{E}$ for the regions $r < a$, $a < r < b$ and $r > b$.
   (c) (5 points) Calculate the potential at the center (relative to infinity).
   (d) (5 points) Calculate the bound volume charge $\rho_b$ in the dielectric layer.
   (e) (5 points) Calculate the bound surface charge $\sigma_b$ for the inner and the outer surface of the dielectric layer.

2. (5 points) Find the magnetic dipole moment of the "bookend-shaped" loop shown in the figure. All sides have length $w$, and it carries a current $I$.



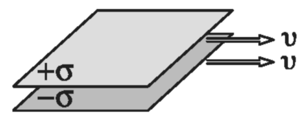

3. (10 points) Draw and explain, qualitatively, the motion of a proton in a combined electric and magnetic field, where the electric field is perpendicular to the magnetic field. The fields are both homogeneous in space and constant in time (static). The proton is initially at rest. For simplicity, you can set the fields along the axes of a cartesian coordinate system: $\mathbf{B} = B_x\hat{\mathbf{x}}$ and $\mathbf{E} = E_z\hat{\mathbf{z}}$, with $B_x$ and $E_z$ positive constants, and the proton initially at the center of the coordinate system.

4. Consider an infinite uniform surface current $\mathbf{K} = K\hat{\mathbf{x}}$, flowing over the $xy$ plane.




(a) (5 points) Use an Amperian loop to find the magnetic field created by this surface current, both above and below the surface.

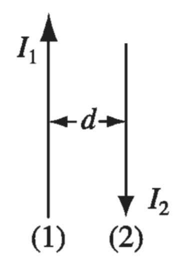
A large parallel-plate capacitor with uniform surface charge $\sigma$ on the upper plate and $-\sigma$ on the lower is moving with constant speed $v$, as is shown in the figure.




(b) (5 points) Find the magnetic field between the plates and also above and below them.

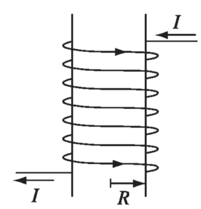
(c) (5 points) Find the magnetic force $F_{\text{mag}} = \int(\mathbf{B} \times \mathbf{A})da$ per unit area on the upper plate, including its direction.

5. Consider two current carrying wires, at a distance $d$, as in the figure on the right.




(a) (5 points) Find the direction and magnitude of the force $F$ per unit length between the two wires.

Now consider a very long solenoid, with $n$ windings per unit length, through which a current $I$ is flowing, as in the figure on the right.




(b) (5 points) Use Amperian loops to show that the magnetic field outside an infinitely long solenoid is zero.

(c) (5 points) Derive the expression for the magnetic field inside the solenoid. Make sure to also give its direction.

(d) (5 points) Now consider the top end of the solenoid, where the magnetic fields leave the solenoid. Explain what paramagnetism is, and explain what the force on a piece of diamagnetic material is above such a solenoid.

## The End